

ROUND ZERO 2025. AF SOLUTIONS

1. A uniform elastic wire of length L and cross-section A is subject to a tensile stress σ , which results in a tensile strain ϵ . Find the work done on the wire.

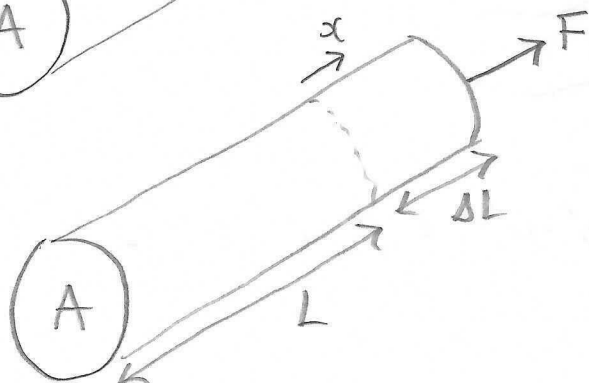
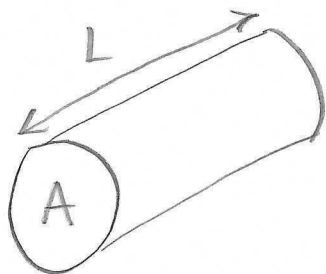
A $LA\sigma\epsilon$

B $\frac{\sigma A}{\epsilon L}$

C $\frac{\sigma L}{2\epsilon A}$

D $\frac{\epsilon A}{\sigma L}$

E $\frac{1}{2}LA\sigma\epsilon$



$$\sigma = \frac{F}{A} \quad \epsilon = \frac{\Delta L}{L}$$

$$\Delta W = \int_0^{\Delta L} F dx = F \Delta L$$

So $\Delta W = \underbrace{A}_{\substack{F \\ \sigma}} \underbrace{\sigma}_{\substack{\epsilon \\ \Delta L}} L$

assuming F is constant. (and σ also).

So this suggests A not E... So F cannot be constant.

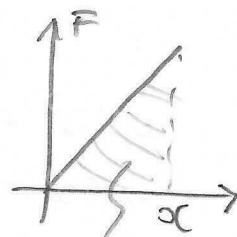
Hooke's Spring, let

$$F = kx$$

$$\Delta W = \frac{1}{2} kx^2$$

Now when $x = \Delta L$, $F = \sigma A$

$$\sigma A = k \Delta L \Rightarrow k = \frac{\sigma A}{\Delta L}$$



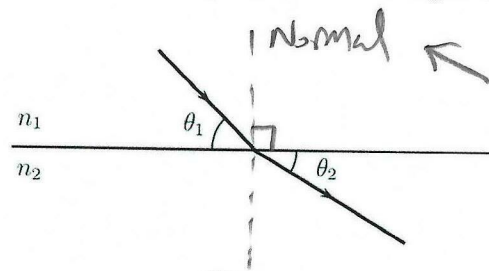
$$\frac{1}{2} Fx = \Delta W$$

$$\therefore \Delta W = \frac{1}{2} \cdot \frac{\sigma A}{\Delta L} (\Delta L)^2 = \frac{1}{2} \sigma A \Delta L$$

$$= \boxed{\frac{1}{2} \sigma A L \epsilon} \text{ as required}$$

So we assume σ is the tensile stress at an extension of ΔL , i.e. strain of ϵ . When $\epsilon = 0$, $\sigma = 0$.

2. The diagram below shows the refraction of a light ray travelling from a medium with refractive index n_1 into a medium with refractive index n_2 . Which equation relates the angles θ_1 and θ_2 ?



← Always start by drawing the normal!

A
 $n_1 \sin \theta_1 = n_2 \sin \theta_2$

B
 $n_1 \sin \theta_2 = n_2 \sin \theta_1$

C
 $n_1 \cos \theta_1 = n_2 \cos \theta_2$

D
 $n_1 \cos \theta_2 = n_2 \cos \theta_1$

E
 $n_1 \sin \theta_1 = n_2 \cos \theta_2$

Snell's law :

$$n_1 \sin(90^\circ - \theta_1) = n_2 \sin(90^\circ - \theta_2)$$

Now $\boxed{\sin(A \pm B)} = \sin A \cos B \pm \cos A \sin(\pm B)$
 $= \boxed{\sin A \cos B \pm \cos A \sin B}$

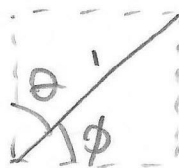
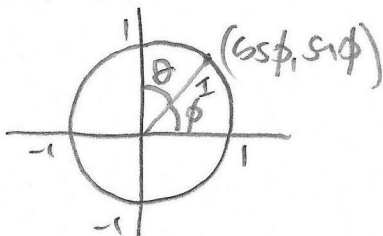
so $\sin(90^\circ - \theta_{1,2}) = \sin 90^\circ \cos \theta_{1,2} - \cos 90^\circ \sin \theta_{1,2}$

$\sin 90^\circ = 1$
 $\cos 90^\circ = 0$

}

$\therefore$

$$\boxed{\sin(90^\circ - \theta_{1,2}) = \cos \theta_{1,2}}$$



$\sin \phi = \cos \theta$

$\therefore$ Snell

$$\Rightarrow \boxed{n_1 \cos \theta_1 = n_2 \cos \theta_2}$$

$\phi = 90^\circ - \theta$ so $\sin(90^\circ - \theta) = \cos \theta$

or use the unit circle.

3. Which of these is **not** a unit of impulse?

A Ns

X

B Ws^2m^{-1}

X

C Pam^3

D kgms^{-1}

X

E CVsm^{-1}

X

Impulse = change of momentum
 $\uparrow I = \Delta(mv)$ so units are $\boxed{\text{kgms}^{-1}}$

Also, since force = rate of change of momentum

$$F = \frac{dp}{dt} \therefore \Delta p = \int F dt$$

so $[\Delta p]$ are $\boxed{\text{Ns}}$ also

$\text{Pa} = \frac{\text{N}}{\text{m}^2}$ so $\frac{\text{N}}{\text{m}^2} \text{m}^3 = \text{Nm}$ which is a Scalar.

$\text{Ws}^2\text{m}^{-1} = \text{Nmms}^{-1} \text{s}^2 \text{m}^{-1}$
 $= \text{Ns}$ so a unit of impulse

$\text{CVsm}^{-1} = C(\text{J/C})\text{s m}^{-1}$
 $= \text{Jsm}^{-1}$
 $= \text{Nm sm}^{-1}$
 $= \text{Ns}$ so a unit of impulse.

4. Uranium-238 ($^{238}_{92}\text{U}$) decays to lead-206 ($^{206}_{82}\text{Pb}$) via a series of alpha and beta minus decays only. How many beta minus decays occur in total?

A 4

B 6

C 8

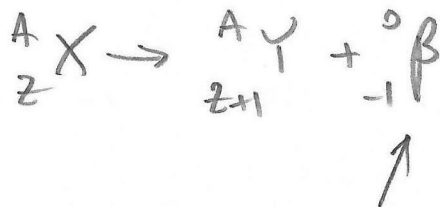
D 10

E 12

α decay: mass #: -4
atomic #: -2



β^- decay: mass #: no change
atomic #: $+1$



$^0_{-1} e^- + ^0_0 \bar{\nu}_e$
↑
electron
anti-neutrino

So

$$\begin{aligned} 238 &= 206 + 4n_\alpha \\ 92 &= 92 - 2n_\alpha + n_\beta \end{aligned}$$

$$\therefore n_\alpha = \frac{238 - 206}{4} = \boxed{8}$$

$$\therefore n_\beta = 92 - 92 + 2 \times 8 = \boxed{6}$$

Same frequency and maintain constant phase difference over time.

5. Two coherent waves of equal amplitude and wavelength 1.8 m, originating from the same source, arrive at the same detector with a path difference of 7.5 m.

Which of these options describes the type of interference that occurs at the detector?

A fully destructive

B mostly destructive

C mostly constructive

D fully constructive

E not enough information

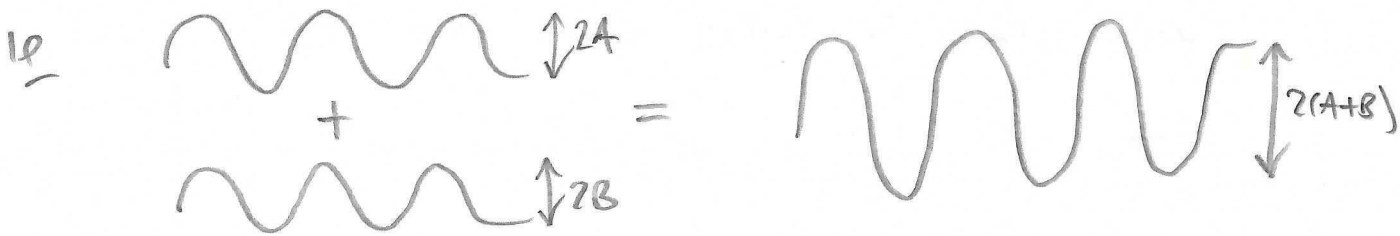
Δx ↓

$$\frac{\text{path difference}}{\lambda} = \frac{7.5}{1.8} = 4.17$$

So Since the ratio above is closer to 4 than 4.5 $\Rightarrow$ "mostly constructive" interference.

if $\Delta x = \text{odd } \# \times \frac{\lambda}{2}$: destructive interference.

if $\Delta x = \text{integer } \# \times \lambda$: constructive interference.



$$3\Omega + 1\Omega = 3\Omega + 3 \times \frac{1}{3}\Omega$$

6. A resistor shop sells individual resistors (the stock is unlimited). The price for each resistor is as follows:

2 Ω for £3, 3 Ω for £1, 6 Ω for £3, 8 Ω for £4, 12 Ω for £2.

What is the least you would need to spend to make a combination of resistors equivalent to exactly 4 Ω ?

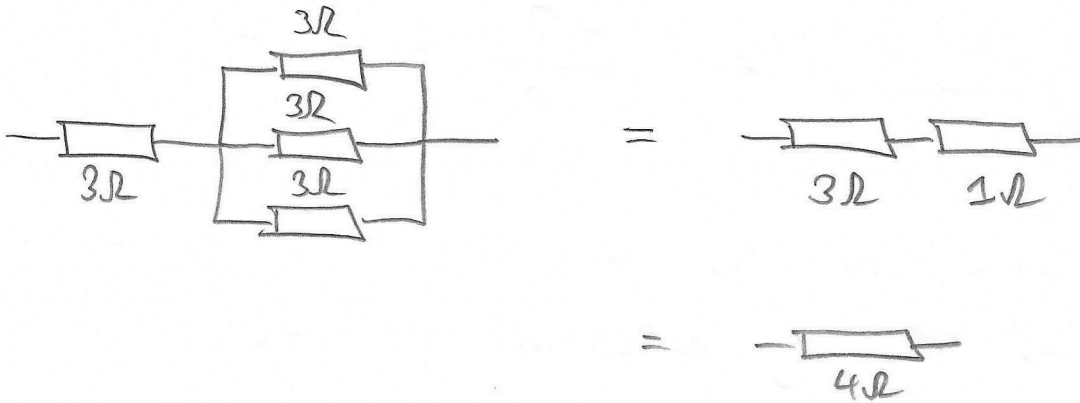
A £3

B £4

C £5

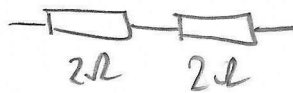
D £6

E £7



So 4 $\times$ 3 Ω resistors are needed, i.e. £4 spend.

Cheaper than

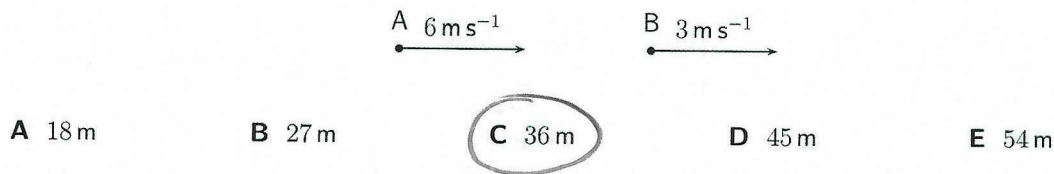


i.e. £6.

Don't need to check others since with price is too high, or // or series combos don't add to 4 Ω

Trick: $4 = 3 + 1$ ↖ can you make 1 (Ω) from a combination? Yes, N resistors of N Ω in parallel.

7. Two particles A & B travel along a straight line at a constant speed with velocities as shown in the diagram. Their initial separation is 18 m. How far does particle A travel before colliding with B?



$$x_A = 6t \rightarrow A$$

$$x_B = 3t + 18 \rightarrow B$$

$$x_A = x_B \text{ when}$$

$$6t = 3t + 18$$

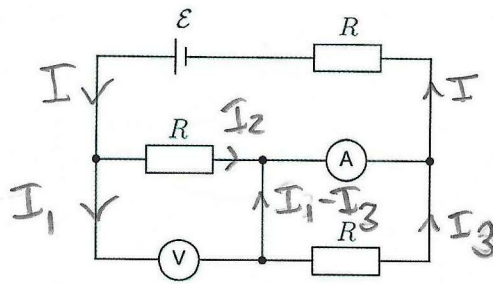
$$\Rightarrow 3t = 18$$

$$t = 6 \text{ (s)}$$

$\therefore$ A travels

$$x_A = 36 \text{ m}$$

8. A cell of emf $\mathcal{E}$ (and negligible internal resistance), is connected to an ideal voltmeter, an ideal ammeter and three resistors R as shown below. What are the readings on the voltmeter and ammeter respectively?



(V) $\approx$ infinite resistance
(A) $\approx$ zero resistance

A $\frac{\mathcal{E}}{3}, \frac{\mathcal{E}}{3R}$

B $\frac{\mathcal{E}}{3}, \frac{\mathcal{E}}{2R}$

C $\frac{\mathcal{E}}{3}, \frac{\mathcal{E}}{R}$

D $\frac{\mathcal{E}}{2}, \frac{\mathcal{E}}{3R}$

E $\frac{\mathcal{E}}{2}, \frac{\mathcal{E}}{2R}$

Resistance of (V) $\rightarrow \infty$ so expect $I_1 \rightarrow 0$

and $\therefore I_2 - I_3$ through (A)

Since $I = I_1 + I_2$, if $I_1 \rightarrow 0 \Rightarrow I_2 \approx I$

so $I - I_3$ through (A).

$V = IR$. (Strictly $I_2 R$, but $I_2 \approx I$).

KVL: $\mathcal{E} = IR + IR$ (top loop)

KVL: $\mathcal{E} = IR + I_3 R + IR$ (lower loop)

$\Rightarrow I_3 = 0$

$\therefore I = \frac{\mathcal{E}}{2R}$

which is the (A) reading

and $V = \frac{\mathcal{E}}{2}$

Quick insight: ignore current through (V), and (A) shorts across lower R , so all current I passes through it: a single loop of $2R$ resistance.

9. Two uniform metal wires of the same length are made from the same material. Separately, the wires are subject to the same force on both ends, and the first harmonic frequency of each is measured.

The frequencies are in the ratio 2 : 1. What is the ratio of their electrical resistances?

[Hint: The speed of a transverse wave on a string is $v = \sqrt{T/\mu}$.]

A 4 : 1

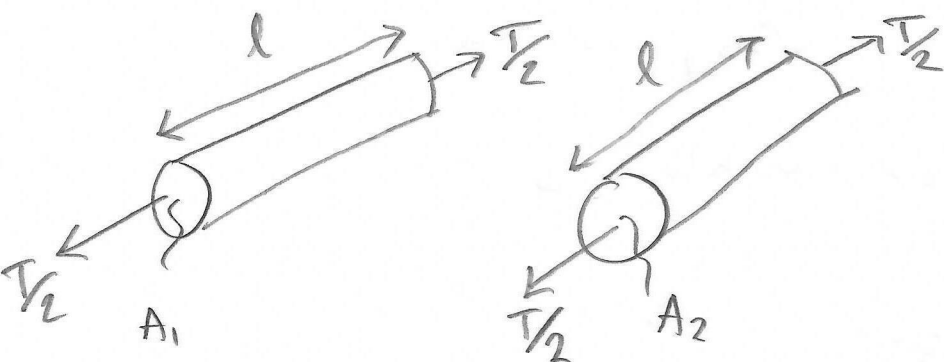
B 3 : 1

C $\sqrt{3} : 1$

D 2 : 1

E $\sqrt{2} : 1$

So difference is the cross sectional area A of each wire



First harmonic means

$$l = \lambda$$

(Fundamental $\Rightarrow l = \lambda/2$)

$$v_{1,2} = f_{1,2} \lambda$$

so if $v_{1,2} = \sqrt{T/\mu}$ and $\mu = \frac{\rho A_{1,2} l}{l} = \rho A_{1,2}$

$$\Rightarrow f_{1,2} l = \sqrt{T/\rho} A_{1,2}^{1/2}$$

so $A_{1,2} = \frac{l^2}{T\rho} f_{1,2}^2$

Now $R_{1,2} = \frac{\rho_E l}{A_{1,2}}$

where ρ_E is resistivity. [Same material so ρ_E same].

$$\therefore \frac{R_1}{R_2} = \frac{A_2}{A_1} = \frac{f_2^2}{f_1^2} = \frac{4}{1}$$

so $\boxed{4:1}$

10. Estimate the number of atoms which make up the Earth.

A 10^{50}

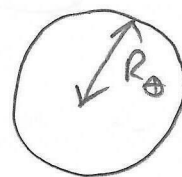
B 10^{55}

C 10^{60}

D 10^{65}

E 10^{70}

$$\begin{array}{ll} \text{Earth radius} & R_{\oplus} \approx 6.37 \times 10^6 \text{ m} \\ \text{Earth mass} & M_{\oplus} \approx 5.97 \times 10^{24} \text{ kg} \end{array}$$



Earth is mostly iron-56, so mass of

$$\approx 56 u = 56 \times 1.66 \times 10^{-27} \text{ kg / atom}$$

$$\therefore \# \text{ atoms in the Earth} \approx \frac{5.97 \times 10^{24}}{56 \times 1.66 \times 10^{-27}}$$

$$\approx 6.4 \times 10^{49}$$

$$\approx \boxed{10^{50}}$$

By mass

or: let atoms be close packed of radius $\approx 10^{-10} \text{ m}$

$$\therefore \# \text{ atoms} \approx \frac{\frac{4}{3} \pi R_{\oplus}^3}{\frac{4}{3} \pi r^3} \approx \left(\frac{R_{\oplus}}{r} \right)^3$$

$$\approx \left(\frac{6.37 \times 10^6}{10^{-10}} \right)^3$$

$$\approx \boxed{2.6 \times 10^{50}}$$

By volume

so 10^{50} to an order of magnitude.